

Quark-Gluon Plasma/Black Hole duality from Gauge/Gravity Correspondence

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Abstract. The Quark-Gluon Plasma (QGP) is the QCD phase of matter expected to be formed at small proper-times in the collision of heavy-ions at high energy. Experimental observations seem to favor a strongly coupled QCD plasma with the hydrodynamic properties of a quasi-perfect fluid, *i.e.* rapid thermalization (or isotropization) and small viscosity. The theoretical investigation of such properties is not obvious, due to the the strong coupling. The Gauge/Gravity correspondence provides a stimulating framework to explore the strong coupling regime of gauge theories using the dual string description. After a brief introduction to Gauge/Gravity duality, and among various existing studies, we focus on challenging problems of QGP hydrodynamics, such as viscosity and thermalization, in terms of gravitational duals of both the static and relativistically evolving plasma. We show how a Black Hole geometry arises naturally from the dual properties of a nearly perfect fluid and explore the lessons and prospects one may draw for actual heavy ion collisions from the Gauge/Gravity duality approach.

1. Introduction

The formation of a QGP (Quark Gluon Plasma) is expected to be realized in high-energy heavy-ion collisions, *e.g.* at RHIC and soon at the LHC. One of the main tools for the description of such a formation is the relevance of relativistic hydrodynamic equations in some intermediate stage of the collisions, see Fig.1. The problem of the hydrodynamic description is the uneasiness

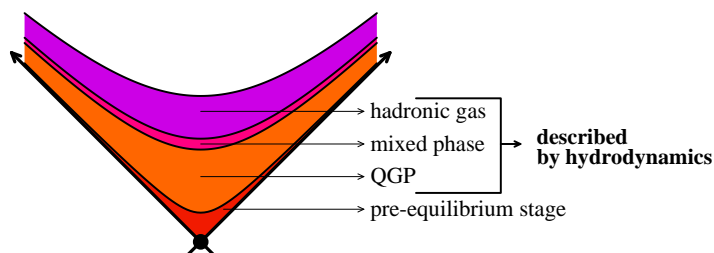


Figure 1. Description of QGP formation in heavy ion collisions.

of the relation with the underlying fundamental theory. Indeed, the experimental observations seem to indicate an almost perfect-fluid behaviour with small shear viscosity, which naturally leads to consider a theory at strong coupling and thus within the yet unknown nonperturbative regime of QCD. Moreover the QGP formation appears to be fast, which may also point towards

strong coupling properties. Another key point of the standard description is the approximate boost-invariance of the process as predicted in the seminal paper of Ref.[1]. The goal of the string theoretic approach is to make use of the gauge/gravity correspondence as a way to tackle the problem of the hydrodynamic behaviour from the fundamental theory point of view. It allows to draw quantitative relations between a strongly coupled gauge field theory and a weakly coupled string theory

More specifically, the AdS/CFT correspondence between the $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory and superstrings in 10 dimensions can be used as a calculational laboratory for this kind of approach, at least as a first stage before a more realistic application to QCD. The unconfined character of the QGP gives some hope that the explicit AdS/CFT example could be useful despite the lack of asymptotic freedom and other aspects specific of QCD.

2. Gauge/Gravity correspondence

As well-known, superstring theory is defined as the quantum embedding of a 2-dimensional world sheet describing the moving string into a 10-dimensional space. In a qualitative way, the gauge-gravity correspondence can be given a geometric interpretation [2]. Considering

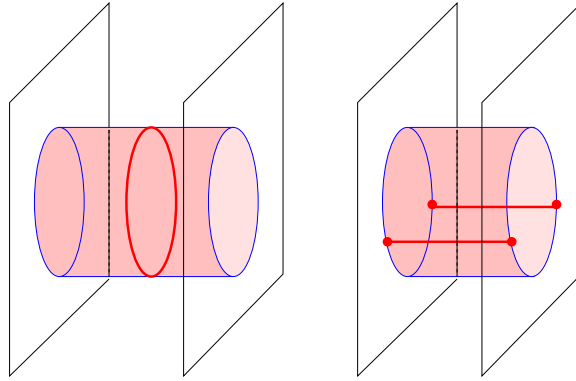


Figure 2. Open \Leftrightarrow closed string duality (from Ref.[2]).

the cylinder stretched between the two *branes* in Fig.2, it may be described as a propagating closed string (Fig.2 left) or an open string with ends on the branes performing a loop (Fig.2 right). In terms of fields, a gravity interaction between branes (left) becomes equivalent to a 1-loop gauge interaction (right). This gauge/gravity *duality* is also a weak/strong coupling one. Indeed, considering a long distance between branes, it corresponds to a weak and thus *classical* gravitational interaction between branes (left), while in gauge theoretical terms, it is expected to imply a long-distance gauge field interaction including many excitation modes of the 1-loop open string, and thus a *quantum* field theory at strong-coupling (right). This duality property can be given a quantitative framework, in particular in the case of AdS/CFT correspondence.

3. Holography and Hydrodynamics

One typical and fascinating aspect of the gauge/gravity duality is the property of *holography*. It states that the amount of information contained in the boundary gauge theory (on the brane) is the same as the one contained in the bulk string theory. In our problem, we shall make use in a quantitative way of this property by taking advantage of one of the remarkable relations due to the holographic renormalization [3]. One can write

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} (= \eta_{\mu\nu}) + z^2 g_{\mu\nu}^{(2)} (= 0) + z^4 \langle T_{\mu\nu} \rangle + z^6 g_{\mu\nu}^{(3)} \dots + ,$$

where $g_{\mu\nu}$ is the bulk metric in 5 dimensions, $\eta_{\mu\nu}$, the boundary metric in physical (3+1) Minkowski space and $\langle T_{\mu\nu} \rangle$, the v.e.v. of the physical energy-momentum tensor. The higher coefficients of the expansion over the fifth dimension z can be obtained by the Einstein equations in the bulk provided the boundary energy-momentum tensor fulfils the zero-trace and continuity equations.

The interesting observation on which we shall elaborate, namely that there is a nontrivial dual relation between a perfect fluid at rest in (3+1) dimensions and a static 5d black hole in the bulk [4] can be proven using holographic renormalization. Indeed, implementing the perfect fluid behaviour with diagonal elements $\langle T_{\mu\nu} \rangle \equiv \text{diag}\{\epsilon, p_1, p_2, p_3\} \propto \text{diag}\{3, 1, 1, 1\}$, one can resum [5] the holographic expansion and get (*via* a change of variables $z \rightarrow \tilde{z}$) the following bulk metric

$$ds^2 = -\frac{1 - \tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2} dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1 - \tilde{z}^4/\tilde{z}_0^4} \frac{d\tilde{z}^2}{\tilde{z}^2} ,$$

where one recognizes the Black Hole (in fact a black brane) with a static horizon at \tilde{z}_0 in the 5th dimension. In fact there exists a one-to-one correspondence between the thermodynamic properties of the Black Hole (BH) and those of the perfect fluid (PF), namely its temperature ($T_{BH} = \epsilon^{\frac{1}{4}} = T_{PF}$) and entropy ($S_{BH} \sim \text{Area} = \epsilon^{\frac{3}{4}} = S_{PF}$).

It is in this context of a static Black hole configuration that one can go further than the perfect fluid approximation and derive the viscosity [6], using the Kubo formula. Indeed, the duality properties extend to a relation between the correlators of the energy-momentum tensor in two space-time points at zero frequency $\omega = 0$ and the absorption cross section of a graviton by the static Black Hole in the bulk. One writes

$$\sigma_{abs}(\omega) \propto \int d^4x \frac{e^{i\omega t}}{\omega} \langle [T_{x_2x_3}(x), T_{x_2x_3}(0)] \rangle \Rightarrow \frac{\eta}{S} \equiv \frac{\sigma_{abs}(0)/16\pi G}{A/4G} = \frac{1}{4\pi} ,$$

where $S = S_{BH} \equiv A/4G$ is the famous entropy-area relation of a Black Hole. From this relation, and putting numbers, it appears that the viscosity is weak, much weaker than the one computed in the weak coupling regime and eventually realizing an absolute viscosity lower bound.

4. QGP and Black Holes: From Statics to Dynamics

The previous results were obtained for static configurations, *i.e.* for a thermalized QGP at rest. In order to take into account, as much as possible, the actual kinematics of a heavy-ion collision, it is required to introduce the proper-time expansion of the plasma. On the gravity side, it calls for studying non-equilibrium geometries, eventually of 5d Black Hole configurations, which represent in itself a nontrivial and interesting issue.

In Ref.[5], it was proposed to build the dual geometries of the standard Bjorken flow [1], that is the description of a boost-invariant expansion of the QGP, which is expected to correspond to the physical situation in the central rapidity region of the collision. In this context the questions why the QGP fluid appears to be nearly perfect (small viscosity) and why its thermalization time can be short have been discussed among other problems.

One starts from a family of proper-time dependent, boundary energy-momentum tensors

$$\langle T_{\mu\nu} \rangle \equiv \text{diag} \left\{ f(\tau) , -\tau^3 \frac{d}{d\tau} f(\tau) - \tau^2 f(\tau) , f(\tau) + \frac{1}{2} \tau \frac{d}{d\tau} f(\tau) , f(\tau) + \frac{1}{2} \tau \frac{d}{d\tau} f(\tau) \right\} ,$$

where the function $f(\tau) \propto \tau^{-s}$ corresponds to an interpolation between different relevant regimes, namely perfect fluid ($p_1 = p_{2,3}$) $s = -\frac{4}{3}$, free streaming ($p_1 = 0$) $s = -1$, fully anisotropic ($p_1 = -p_{2,3}$) $s = 0$. Using the holographic renormalization to compute the coefficients of the corresponding metrics in the expansion on the fifth dimension and after resummation, it was possible to solve the dual geometry for given s at asymptotic proper-time τ . It reveals the

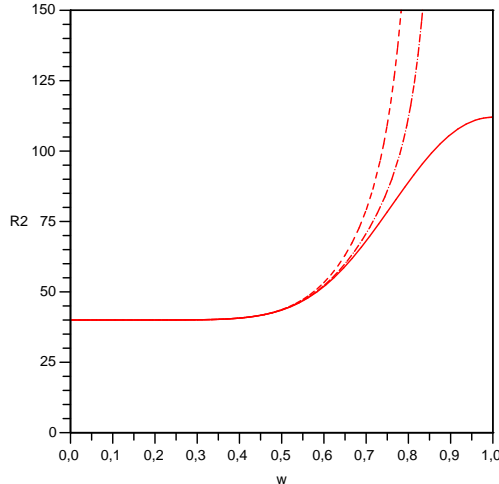


Figure 3. The curvature scalar \mathcal{R}^2 for $s = \frac{4}{3} \pm 1$: Nonsingular Geometry \Leftrightarrow Perfect Fluid.

existence of a scaling property of the solutions in terms of the proper-time dependent variable $v = \frac{z}{\tau^{1/3}}$.

Analyzing the family of solutions as a function of s , it appears that the only nonsingular solution for invariant scalar quantities (here the square of the Ricci tensor $\mathcal{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$, see Fig.3), is obtained for $s = 4/3$. This solution is the only one of the family corresponding to a Black Hole moving away in the fifth dimension. Hence the perfect-fluid case is singled out and the moving Black Hole in the bulk corresponds through duality to the expansion of the QGP taking place in the boundary. Consequently, the BH horizon moves as $z_h(\tau) \propto \tau^{\frac{1}{3}}$, the temperature as $T(\tau) \sim 1/z_h \sim \tau^{-\frac{1}{3}}$, and the entropy stays constant since $S(\tau) \sim Area \sim \tau \cdot 1/z_h^3 \sim const.$ Note that again the physical thermodynamical variables of the QGP are the same as those one may attribute to the BH in the bulk (with the reservation that thermodynamics of a moving BH may rise nontrivial interpretation problems). Hence one finds a concrete realization of the idea [7] of a duality between the QGP formation and a moving a moving Black Hole.

5. Thermalization

There has been a lot of activity along the lines of the AdS/CFT correspondence and its extensions to various geometric configurations. Dual studies of quark energy loss [8], jet quenching [9], quark dragging [10], etc... have been and are being performed. Sticking to the configurations corresponding to an expanding plasma and going beyond the first order terms in proper-time, one has obtained results [11] on the viscosity, confirming the universal bound of Ref.[6], on the relaxation time of the plasma and very recently on the inclusion of flavor degrees of freedom [12].

Let us finally focus on the thermalization problem, which can be usefully taken up using the gauge/gravity duality in the strong coupling hypothesis. the problem is to give an explanation to the strikingly small thermalization time required for the formation of a QGP as can be abstracted from the experimental observations. Already in ref.[13], it has been found that if one performs a small deviation from the Black hole metric by coupling with a scalar field and analyze the so-called quasi-normal modes defining the way how the system relaxes to its initial state,

one finds a numerically small value of the relaxation time in units of the local (and evolving) temperature. Even if a definite value of this relaxation time cannot be inferred at this stage due to scale-invariance, this result was suggestive of a stability of the QGP in the strong coupling regime with respect to perturbations out of equilibrium.

In order to go further, one has to deal with the problem of the QGP evolution at small proper-times. In Ref[14], the holographic renormalization program has been pursued for the small proper-time expansion. Relaxing the selection of the appropriate metric by requiring only the metric tensor to be a real and single-valued function of the coordinates everywhere in the bulk, one finds an unique solution corresponding to the “fully anisotropic case”, *i.e.* $s = 0$. In the same

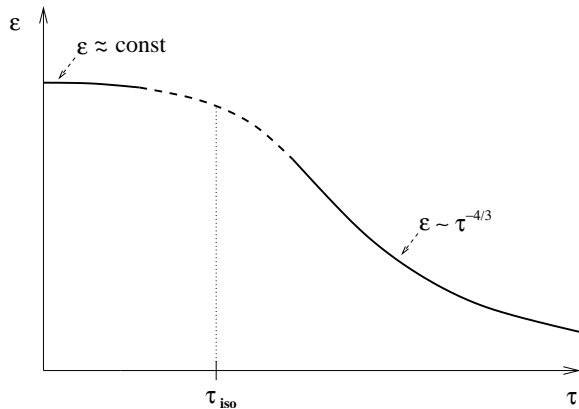


Figure 4. Evaluation of the isotropization/thermalization time (from Ref.[14]).

paper, an evaluation of the range of the isotropization time has been proposed, by extrapolation of realistic estimates abstracted from experiments to the supersymmetric case. The idea is to match the large and small proper-time regimes at some value of the proper-time τ_{iso} . This proper-time is mathematically defined as the crossing value for the branch-point singularities of both regimes. Physically, it is expected to give an estimate of the proper-time range during which the medium evolves from the full anisotropic regime (small τ) to the perfect fluid one (large τ). In order to give an idea of the possible physical implications of this strong coupling scheme, let us shortly reproduce the estimate made in Ref.[14]. Implementing the estimated physical value of the energy density at some proper-time (*e.g.* $\epsilon(\tau) = e_0 \tau^{4/3}|_{\tau=.6} \sim 15 \text{ GeV fermi}^{-3}$) one finds

$$\tau_{iso} = \left(\frac{3N_c^2}{2\pi^2 e_0} \right)^{3/8} \sim .3 \text{ fermi} .$$

This short isotropization time thus seems a characteristic feature of the strong coupling scenarios. It is clear that more realistic estimates should take into account less idealized dual models, corresponding to QCD, such as the lack of supersymmetry and the finite numbers of colors. However, the non confined character of the QGP and the robustness of some predictions (such as the η/S ratio) may give some confidence that this short isotropization time could be a reasonable estimate at strong coupling.

6. Conclusions and outlook

From the present rapid (and partial) survey of some of the results obtained in the AdS/CFT approach to the formation and expansion of the Quark-Gluon plasma in heavy-ion collisions, it appears that the gauge/gravity correspondence is a promising way to explore some features of QCD at strong coupling. Indeed some general features of this correspondence, relating at long

distances the closed and open string geometries (see Fig.2) are expected to be valid in principle for various dual schemes and thus, hopefully, QCD.

In practice, the quantitative dual schemes have been more precisely elaborated for the specific AdS/CFT case, *i.e.* the gauge theory with $\mathcal{N} = 4$ supersymmetries. Among the results, it gives a calculable link between the hydrodynamic quasi-perfect fluid behaviour on the gauge theory side with a BH geometry in the higher dimensional gravity side in an AdS background. This relation can be extended from the static case to a dynamical regime reflecting (within the AdS/CFT framework) the relativistic expansion of the corresponding quark-gluon plasma. This, and many other applications, some of them using more complex geometries, less supersymmetric backgrounds and examining other observables, gives hope on the fruitful possibilities of the gauge/gravity approach to the QGP formation.

As an outlook, it is worth mentioning some of the possible new directions of study one is led to consider. Starting with the more technical ones, it is known that the Bjorken flow is not exactly verified in heavy-ion collisions, since the observed distribution of particles is nearly gaussian in rapidity and thus not reflecting exactly the boost-invariance of the Bjorken flow. It would be interesting to investigate dual properties for non-boost invariant flows, such as the Landau solution [15]. On a more general ground, the whole approach still concerns only the hydrodynamical stage of the QGP expansion. It would be important to attack both the initial (partonic) and final (hadronic) stages of the reaction in the same framework and thus the problem of *phase transitions* during the collision. Finally, one would like to have more realistic dual frameworks including a finite number of colors, flavor degrees of freedom and no (or broken) supersymmetry.

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